

Cosmic Ray Acceleration at Relativistic Shock Waves

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Abstract

Theory of the first-order Fermi acceleration of cosmic ray particles at relativistic shock waves is reviewed. We consider shocks with parallel and oblique, sub- and super-luminal magnetic field configurations and with finite-amplitude magnetic field perturbations. A role of oblique field configurations and field perturbations in forming the cosmic ray energy spectrum and changing the acceleration time scale is discussed.

1. Particle acceleration at non-relativistic shock waves

Processes of first-order particle acceleration at non-relativistic shock waves were widely discussed by a number of authors during the last two decades (for review, see, e.g. Drury (1983), Blandford & Eichler (1987), Berezhko et al. (1988), Jones & Ellison (1991)). Below, we remind the basic physical picture and some most of the important results obtained within this theory, to be later compared with the results obtained for relativistic shocks.

The preferred by us simple description of the acceleration process consists in considering two plasma rest frames, the *upstream frame* and the *downstream one*. We use indices ‘1’ or ‘2’ to indicate quantities measured in the upstream or the downstream frame respectively. If one neglects the second-order Fermi acceleration, the particle energy is a constant of motion in any plasma rest frame and energy changes occur when the particle momentum is Lorentz-transformed at each crossing of the shock. In the case of *parallel* shock, with the mean magnetic field parallel to the shock normal, the acceleration of an individual particle is due to a consecutive shock crossings by the diffusively wandering particle. Each *upstream-downstream-upstream* diffusive loop results in a small increment of particle momentum, $\Delta p \propto U_1/v$, where v is the particle velocity and U_1 is the shock velocity in the upstream frame, $U_1 \ll v \approx c$. In oblique shocks, the particle helical trajectories can cross the shock surface a number of times at any individual shock transition or reflection.

The most interesting feature of the first-order Fermi acceleration at a non-relativistic plane-parallel shock wave is independence of the *test-particle stationary* particle energy spectrum from the background conditions near the shock, including the mean magnetic field configuration and the spectrum of MHD turbulence. The main reason behind that is a nearly-isotropic form of the particle momentum distribution at the shock. If a sufficient amount of scattering occurs near the shock, this condition always holds for the shock velocity along the upstream magnetic field $U_{B,1} \equiv U_1 / \cos \Psi_1 \ll v$ (Ψ_1 - the upstream magnetic field inclination to the shock normal). Independently of the field inclination at the shock, the particle density is continuous across it and the spectral index for the phase-space distribution function, α , is defined exclusively in the terms of the shock compression ratio R :

$$\alpha = \frac{3R}{R-1} \quad . \quad (1.1)$$

Because of the isotropic form of the particle distribution function, the spatial diffusion equation has become a widely used mathematical tool for describing particle transport and acceleration processes in non-relativistic flows. The characteristic acceleration time scale at the parallel shock is

$$T_{acc} = \frac{3}{U_1 - U_2} \left\{ \frac{\kappa_1}{U_1} + \frac{\kappa_2}{U_2} \right\} \quad , \quad (1.2)$$

where $\kappa_i \equiv \kappa_{\parallel,i}$ is the respective particle spatial diffusion coefficient along the magnetic field, as discussed by e.g. Lagage & Cesarsky (1983). Ostrowski (1988, see also Bednarz & Ostrowski 1996) derived an analogous expression for shocks with oblique magnetic fields and small amplitude magnetic field perturbations. For a negligible cross-field diffusion and for $U_{B,1} \ll c$ it can be written in essentially the same form as the one given in Equ. (1.2), with all quantities taken as the normal (n) ones with respect to the shock ($\kappa_{n,i}$ for κ_i ($i = 1, 2$)). Usually $\kappa_n < \kappa_{\parallel}$. Therefore, as confirmed by measurements in the heliosphere, the oblique shocks may be much more rapid accelerators as compared to the parallel shocks.

2. Cosmic ray acceleration at relativistic shock waves

2.1 The Fokker-Planck description of the acceleration process

In the case of the shock velocity (or its projection $U_{B,1}$) reaching values comparable to the light velocity, the particle distribution at the shock becomes anisotropic. This fact complicates to a great extent both the physical picture and the mathematical description of particle acceleration. The first attempt to consider the acceleration process at the relativistic shock was presented in 1981 by Peacock (see also Webb 1985); however, no consistent theory was proposed until a paper of Kirk & Schneider (1987a; see also Kirk 1988) appeared. Those authors considered the stationary solutions of the relativistic Fokker-Planck equation for particle pitch-angle diffusion for the case of parallel shock wave. In the situation with the

gyro-phase averaged distribution $f(p, \mu, z)$, which depends only on the unique spatial co-ordinate z along the shock velocity, and with μ being the pitch-angle cosine, the equation takes the form

$$\Gamma(U + v\mu) \frac{\partial f}{\partial z} = C(f) + S \quad , \quad (2.1)$$

where $\Gamma \equiv 1/\sqrt{1-U^2}$ is the flow Lorentz factor, $C(f)$ is the collision operator and S is the source function. In the presented approach, the spatial co-ordinates are measured in the shock rest frame, while the particle momentum co-ordinates and the collision operator are given in the respective plasma rest frame. For the applied pitch-angle diffusion operator, $C = \partial/\partial\mu(D_{\mu,\mu}\partial f/\partial\mu)$, they generalised the diffusive approach to higher order terms in particle distribution anisotropy and constructed general solutions at both sides of the shock which involved solutions of the eigenvalue problem. By matching two solutions at the shock, the spectral index of the resulting power-law particle distribution can be found by taking into account a sufficiently large number of eigenfunctions. The same procedure yields the particle angular distribution and the spatial density distribution. The low-order truncation in this approach corresponds to the standard diffusion approximation and to a somewhat more general method described by Peacock. The above analytic approach (or the ‘semi-analytic’ one, as the mentioned matching of two series involves numerical fitting of the respective coefficients) was verified by Kirk & Schneider (1987b) by the method of particle Monte Carlo simulations.

An application of this approach to more realistic conditions – but still for parallel shocks – was presented by Heavens & Drury (1988), who investigated the fluid dynamics of relativistic shocks (cf. also Ellison & Reynolds 1991) and used the results to calculate spectral indices for accelerated particles (Fig. 1). They considered the parallel shock wave propagating into electron-proton or electron-positron plasma, and performed calculations using the analytic method of Kirk & Schneider for two different power spectra for the scattering MHD waves. In contrast to the non-relativistic case, they found (see also Kirk 1988) that the particle spectral index depends on the form of the wave spectrum. An interesting fact was revealed that *synchrotron* spectral indices $\gamma (\equiv (\alpha - 3)/2)$ obtained for shock velocities ranging from non-relativistic ones up to $U_1 = 0.98 c$ fell into a relatively narrow gap, between 0.35 and 0.6 for both the fluids considered. They also noted a strange fact that the non-relativistic expression (1.1) provided a quite reasonable approximation to the actual spectral index.

A substantial progress in understanding the acceleration process in the presence of highly anisotropic particle distributions is due to the work of Kirk & Heavens (1989; see also a discussion in Ostrowski 1991a and Ballard & Heavens 1991), who considered particle acceleration at *subluminal* ($U_{B,1} < c$) relativistic shocks with oblique magnetic fields. They assumed the magnetic momentum conservation, $p_\perp^2/B = \text{const}$, at particle interaction with the shock and applied the Fokker-Planck equation discussed above to describe particle transport along the field lines outside the shock. Within the considered approach, a possibility of cross-field diffusion is excluded. In the cases when $U_{B,1}$ reached relativistic values, they derived very

Fig. 1 The particle spectral indices α at parallel shock waves propagating in the cold (e, p) plasma versus the shock velocity U_1 (Heavens & Drury 1988). On the right vertical axis the respective synchrotron spectral index γ is given. Using the solid line and the dashed line (long dashes) we show indices for two choices of the turbulence spectrum. The dashed line with short dashes gives the spectral index derived formally from the non-relativistic Equ. 1.1. The horizontal line $\alpha = 4.0$ is given for the reference purposes.

flat energy spectra with $\gamma \approx 0$ at $U_{B,1} \approx 1$ (Fig. 2). In such conditions particle density in front of the shock can substantially - even by a few orders of magnitude - exceed the downstream density (see the curve denoted ‘-8.9’ at Fig. 3). Creating flat spectra and great density contrasts is due to effective reflections of anisotropically distributed upstream particles from the region of compressed magnetic field downstream the shock. However, the conditions leading to very flat spectra are supposed to be accompanied by processes - like a large amplitude wave generation upstream the shock - leading to the spectrum steepening (Ostrowski 1991a; see also Sec. 2.3).

As stressed by Begelman & Kirk (1990), in relativistic shocks one can often find the superluminal conditions, with $U_{B,1} > c$, where the above presented approach is no longer valid. Then, it is not possible to reflect upstream particles from the shock and to transmit downstream particles into the upstream region. In effect, only a single compression of the transmitted particles’ helical orbits re-shapes the upstream particle distribution by shifting particle energies to larger values. Begelman & Kirk show that the energy gains in such a process can be quite significant, exceeding the value expected for the adiabatic compression. The physical reason behind that is the growing particle anisotropy during compression at the shock, providing the excessive pressure along the shock normal.

The approach proposed by Kirk & Schneider (1987a) and the derivations of Begelman & Kirk (1990) are valid only in the case of weakly disturbed background

Fig. 2 Spectral indices α of particles accelerated at oblique shocks versus shock velocity projected at the mean magnetic field, $U_{B,1}$. On the right vertical axis the respective synchrotron spectral index γ is given. The shock velocities U_1 are given near the respective curves taken from Kirk & Heavens (1989). The points were taken from simulations deriving explicitly the details of particle-shock interactions (Ostrowski 1991a). The results are presented for compression $R = 4$.

magnetic fields. However, in the efficiently accelerating shocks one may expect the large amplitude waves to be present, when both the Fokker-Planck approach is no longer valid and the magnetic momentum conservation no longer holds for oblique shocks (Ostrowski 1991a). In such a case, numerical methods have to be used.

2.2 Particle acceleration in the presence of large amplitude magnetic field perturbations

The first attempt to consider the acceleration process at parallel shock wave propagating in a turbulent medium was presented by Kirk & Schneider (1988), who included into Equ. 2.1 the Boltzmann collision operator describing the large angle scattering. By solving the resulting integro-differential equation with the use of analytic means they demonstrated hardening of the particle spectrum with increasing contribution of the large-angle scattering. The reason for such a spectral change is the additional isotropization of particles interacting with the shock, leading to the increased particle mean energy gain. In oblique shocks, this simplified approach can not be used, not only because of the distribution anisotropy, but also due to the fact that the character of individual particle-shock interaction - reflection and transmission characteristics - depends on magnetic field perturbations. Let us additionally note that application of the point-like large angle scattering model in relativistic shocks does not provide a viable physical representation of the scattering at MHD waves (Bednarz & Ostrowski 1996).

Fig. 3 The energetic particle density across the relativistic shock with oblique magnetic field. The shock with $U_1 = 0.5$, $R = 5.11$ and $\psi_1 = 55^\circ$ is considered. The curves for different perturbation amplitudes are characterized with the value $\log \kappa_\perp / \kappa_\parallel$ given near the curve. The data are vertically shifted for picture clarity. The value X_{max} is the distance from the shock at which the upstream particle density decreases to 10^{-3} part of the shock value.

To handle the problem of particle spectrum in a wide range of background conditions the particle Monte Carlo simulations were proposed (Kirk & Schneider 1987b; Ellison et al. 1990; Ostrowski 1991a, 1993; Ballard & Heavens 1992). At first, let us consider subluminal shocks. The field perturbations influence the acceleration process in various ways. As they enable the particle cross field diffusion, a modification (decrease) of the downstream particle's escape probability may occur. This factor tends to harden the spectrum. Next, the perturbations decrease particle anisotropy, leading to increase of the mean energy gain of reflected upstream particles, but - which is more important for oblique shocks - this increases also the particle downstream transmission probability, enabling them to escape efficiently from the further acceleration. The third factor is due to perturbing particle trajectory during an individual interaction with the shock discontinuity and breakdown of the approximate conservation of p_\perp^2/B . Because reflecting a particle from the shock requires a fine tuning of particle trajectory with respect to the shock surface, even a very small amount of perturbations can decrease the reflection probability in a substantial way. The influence at the acceleration time scale is discussed in the next section. Simulations show (Fig. 4) that - until the wave amplitude becomes very large - factors leading to more efficient particle escape are dominant and may result in steepening of the spectrum to $\gamma \sim 0.5 - 0.8$ may occur. At extremely large wave amplitudes, a slight flattening of the spectrum may take place. As presented at Fig. 3, the increased transmission probability for upstream particles leads also to lowering the cosmic ray density contrast across the shock (Ostrowski 1991b).

Fig. 4 Spectral indices for oblique relativistic shocks versus perturbation amplitude $\delta B/B$ (Ostrowski 1993). Different field inclinations are characterized by values of $U_{B,1}$ given near the respective results, $U_{B,1} < 1$ for subluminal shocks and $U_{B,1} \geq 1$ for superluminal ones. Absence of data for small field amplitudes in superluminal shocks is due to no-power-law character of the spectrum or extremely steep power law spectra occurring in these conditions.

In parallel shock waves propagating in a highly turbulent medium the effects discovered for oblique shocks can also manifest because of the *local* perturbed magnetic field compression at the shock. The problem was considered using the technique of particle simulations by Ballard & Heavens (1992). They showed a possibility to have very steep spectra in this case, with the spectral index growing from $\gamma \sim 0.6$ at medium relativistic velocities up to nearly 2.0 at $U_1 = 0.98$. These results apparently do not correspond to large-perturbation-amplitude limit of Ostrowski's (1993) simulations for oblique shocks and the analytic results of Heavens & Drury (1988). In the paper of Ostrowski possible reasons for the discrepancy are discussed.

For large amplitude magnetic field perturbations the acceleration process in the super-luminal shocks can lead to the power-law particle spectrum formation, against the statements of Begelman & Kirk (1990) valid at small wave amplitudes only. Such general case was discussed by Ostrowski (1993) (Fig. 4) and by Bednarz & Ostrowski (1996, see below) using the method of particle simulations.

2.3 The acceleration time scale

The shock waves propagating with relativistic velocities raise also interesting questions pertaining to the cosmic ray acceleration time scale, T_{acc} . A simple comparison to the non-relativistic formula based on numerical simulations shows that T_{acc} relatively decreases with increasing shock velocity for parallel (Quenby

Fig. 5 The acceleration time T_{acc} versus the level of particle scattering measured by the ratio of $\kappa_{\perp}/\kappa_{\parallel}$ for the shock with velocity $U_1 = 0.5$. We present results for three values of the magnetic field inclination: a.) parallel shock ($\psi_1 = 1^\circ$), b.) a sub-luminal shock with $\psi_1 = 45.6^\circ$ and c.) a super-luminal shock with $\psi_1 = 89^\circ$. The *maximum* value of the model parameter Δt – corresponding to the small wave amplitude limit – is given at the end of each curve and the wave amplitude monotonously increases along each curve up to $\delta B \sim B$. $r_{e,1}$ is the particle gyroradius in the effective (including perturbations) upstream magnetic field (cf. Bednarz & Ostrowski 1996).

& Lieu 1989; Ellison et al. 1990) and oblique (Takahara & Terasawa 1990; Newman et al. 1992; Lieu et al. 1994; Quenby & Drolas 1995; Naito & Takahara 1995) shocks. However, the numerical approaches used there, based on assuming particle isotropization for all scatterings, neglect or underestimate a rather significant factor affecting the acceleration process – the particle anisotropy. Ellison et al. (1990) and Naito & Takahara (1995) included also the more realistic, in our opinion, derivations involving the pitch-angle diffusion approach. The calculations of Ellison et al. for parallel shocks show similar results to those they obtained for large amplitude scattering. In the shock with velocity $0.98c$ the acceleration time scale is reduced by the factor ~ 3 with respect to the non-relativistic formula of Equ. 1.2. Naito & Takahara considered shocks with oblique magnetic fields. They confirmed reduction of the acceleration time scale with increasing inclination of the magnetic field, derived earlier for non-relativistic shocks. However, their approach neglected effects of particle cross field diffusion and assumed the adiabatic invariant conservation in particle interactions with the shock. These two simplifications limit validity of their results to the cases with small amplitude turbulence near the shock.

A much wider discussion of the acceleration time is provided recently by Bednarz & Ostrowski (1996), who apply numerical simulations involving the small

Fig. 6 The values of T_{acc} in units of $r_{e,1}/c$ at different inclinations ψ_1 versus the particle spectral index α . The values resulting from simulations are presented for $U_1 = 0.5$ for five values of the angle ψ_1 given near the respective results. The *maximum* value of the model parameter Δt – corresponding to the small wave amplitude limit – is given at the end of each curve and the wave amplitude monotonously increases along each curve up to $\delta B \sim B$; $r_{e,1}$ - see Fig. 5.

angle particle momentum scattering (Ostrowski 1991a). However, the approach is believed to provide also a reasonable description of particle transport in the presence of large amplitude magnetic field perturbations, and thus to enable modeling the effects of cross-field diffusion. The authors suggest that due to noticeable correlations present in the acceleration process, any derivation of the acceleration time scale cannot use the distribution of energy gains and the distribution of times between successive particle-shock interactions separately. T_{acc} is defined as the time scale describing the rate of change of the cut-off energy in the time dependent particle spectrum evolution. The results are presented for shock waves with parallel and oblique (both, sub- and super-luminal) magnetic field configurations, and field perturbations with amplitudes ranging from very small ones up to $\delta B \sim B$ (Fig-s 5,6). In all cases the values of T_{acc} are given in the shock *normal* rest frame. In parallel shocks T_{acc} diminishes with the growing perturbation amplitude (Fig. 5) and the shock velocity U_1 . However, it is approximately constant for a given value of U_1 if we use the formal diffusive time scale, $\kappa_1/(U_1 c) + \kappa_2/(U_2 c)$, as the time unit. Another qualitative feature discovered in oblique shocks is that due to the cross-field diffusion T_{acc} can change with δB in a non-monotonic way (Fig. 5). The acceleration process leading to the power-law spectrum is possible in super-luminal shocks only in the presence of large amplitude turbulence. Then, in contrast to the quasi-parallel shocks, T_{acc} increases with increasing δB . In some cases with the oblique magnetic field configurations one may note a possibility to have an

extremely short acceleration time scales comparable to the particle gyroperiod in the magnetic field upstream the shock. A coupling between the acceleration time scale and the particle spectral index is presented in Fig. 6. One should note that the form of involved relation is contingent to a great extent on the magnetic field configuration.

3. Final remarks

Substantial amount of work done to date on the *test particle* cosmic ray acceleration at relativistic shocks yielded not too promising results for meaningful modeling of the observed astrophysical radiation sources. The main reason for that deficiency is - in contrast to non-relativistic shocks - a direct dependence of the derived spectra on the conditions at the shock. Not only the shock compression ratio, but also other parameters, like the mean inclination of the magnetic field or the wave spectrum shape and amplitude, are significant here. Depending on the actual conditions one may obtain spectral indices as flat as $\alpha = 3.0$ ($\gamma = 0.0$) or very steep ones, $\alpha > 5.0$ ($\gamma > 1.0$). The background conditions leading to the very flat spectra are probably subject to some instabilities; however, there is no detailed derivation describing the instability growth and the resulting cosmic ray spectrum modification.

It seems that a true progress in modeling particle acceleration in actual sources requires a full non-linear description, including feedback of accelerated particles at the turbulent wave fields near the shock wave, flow modification caused by the cosmic rays' plasma pre-shock compression and, of course, the appropriate boundary conditions. A simple approach to the parallel shock case was presented by Baring & Kirk (1991), who found that relativistic shocks could be very efficient accelerators. However, it seems to us that in a more general case it will be very difficult to make any substantial progress in that matter. The difficulty arises here from the fact that the generated energetic particle spectrum (and the corresponding particle pressure) depends on the number of parameters, and any choice of these parameters can substantially affect the resulting plasma flow pattern across the shock and the particle spectrum. Moreover, for very flat spectra obtained at the shock the non-linear acceleration picture depends to a large extent on the detailed knowledge of the background and boundary conditions in the scales relevant for particles near the upper energy cut-off. The existence of stationary solutions is doubtful in this case.

One may note that observations of possible sites of relativistic shock waves (knots and hot spots in extragalactic radio sources), which allow for determination of the energetic electron spectra, often yield particle spectral indices close to $\alpha = 4.0$ ($\gamma = 0.5$). In order to overcome difficulties in accounting for these data Ostrowski (1994a) proposed an additional '*law of nature*' for non-linear cosmic ray accelerators. The particles within different energy ranges do not couple directly with each other and are supposed to form independent 'degrees of freedom' in the system. Our 'law' provides that the nature prefers energy equipartition between such degrees of freedom, yielding the spectra with $\alpha \approx 4.0$.

Finally, let us mention an independent possibility of particle acceleration in relativistic flows at *non-compressive* tangential discontinuities or shear layers (see Ostrowski in the present volume). This interesting possibility has not been adequately discussed to date.

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